## QUIZ 19 SOLUTIONS: LESSON 24
## MARCH 22, 2019

Write legibly, clearly indicate the question you are answering, and put a box or circle around your final answer. If you do not clearly indicate the question numbers, I will take off points. Write as much work as you need to demonstrate to me that you understand the concepts involved. If you have any questions, raise your hand and I will come over to you.

1. [10 pts] A biologist must make a medium to grow a type of bacteria. The percentage of salt in the medium is given by

$$S = 0.01x^2y^2z$$

where $S$ is the percentage expressed as a decimal.

$x$, $y$, $z$ are amounts in liters of 3 different nutrients mixed together to create the medium. The ideal salt percentage for this type of bacteria is 40%. The costs of $x$, $y$, $z$ are 1, 2, 3 dollars per liter.

Determine the minimum cost that can be achieved. Round your answer to the 4 decimal places.

We want to minimize Cost subject to $.40 = .01 \, x^2y^2z$ (since our goal is to achieve the proper salt percentage).

$$\text{Cost} = C(x,y,z) = x + 2y + 3z$$

Since $.40 = .01 \, x^2y^2z \implies 40 = x^2y^2z \implies z = \frac{40}{x^2y^2}$, we substitute

$$\text{Cost} = C(x,y) = x + 2y + 3\left(\frac{40}{x^2y^2}\right) = x + 2y + \frac{120}{x^2y^2}$$

Next, we find the critical points of C:

$$0 = C_x = 1 - \frac{2 \cdot (120)}{x^3y^2} \qquad\qquad 0 = C_y = 2 - \frac{2 \cdot (120)}{x^2y^3}$$

$$0 = C_x = 1 - \frac{240}{x^3y^2} \qquad\qquad 0 = C_y = 2 - \frac{240}{x^2y^3}$$

$$\implies 1 = \frac{240}{x^3y^2} \qquad\qquad \implies 2 = \frac{240}{x^2y^3}$$

$\Rightarrow \quad x^3 y^2 = 240$ $\qquad \Rightarrow x^2 y^3 = \frac{1}{2}(240)$

Now, $x, y \neq 0$ and

$$x^2 y^3 = \frac{1}{2}(240) = \frac{1}{2}(x^3 y^2) \qquad \text{since} \quad x^3 y^2 = 240$$

So, $x^2 y^3 = \frac{1}{2} x^3 y^2$. Dividing both sides through by $x^2 y^2$,

$$y = \frac{1}{2} x$$

Since $x^3 y^2 = 240$:

$$240 = x^3 \left(\frac{1}{2}x\right)^2 = x^3 \left(\frac{1}{4}x^2\right) = \frac{1}{4} x^5$$

$$\Rightarrow \quad 960 = x^5 \quad \Rightarrow \quad x = \sqrt[5]{960}$$

Then $y = \frac{1}{2} \sqrt[5]{960}$

Our minimum cost is then

$$C\left(\sqrt[5]{960}, \frac{1}{2}\sqrt[5]{960}\right) = \sqrt[5]{960} + 2\left(\frac{1}{2}\sqrt[5]{960}\right) + \frac{120}{\left(\sqrt[5]{960}\right)^2 \left(\frac{1}{2}\sqrt[5]{960}\right)^2}$$

$$\approx \boxed{\$9.8718}$$